MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

DEM5028 – ENGINEERING MATHEMATICS 2

(Diploma in Electronic Engineering)

22 OCTOBER 2019
9.00 AM – 11.00 AM
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages excluding the cover page with **4** questions only.
2. Attempt **ALL FOUR** questions.
3. Write your answers in the answer booklet provided.
4. Formulae are provided in the appendix section.

QUESTION 1

a. Evaluate the following integrals.

i. $\int \frac{3x^3 - 8x^2 + 4x - 1}{x^2 - 3x + 2} dx$ (6 marks)

ii. $\int_0^{\frac{\pi}{4}} x^2 \sin 2x \, dx$ (6 marks)

b. Find the area of the region bounded by the curve $f(x) = x(x-2)(x-5)$ and the x-axis. (7 marks)

c. Find the volume of the solid obtained by revolving the region bounded by the curve $y = \frac{x^3}{4}$ and the line $y = x$ about the y-axis. Consider the region in the first quadrant only. (6 marks)

[TOTAL 25 MARKS]

QUESTION 2

a. Solve the differential equation $\frac{dy}{dx} = e^{-y}(2x - 4)$ where $y(5) = 0$. Hence, find the value of y if $x = 10$. (11 marks)

b. Find the solution of the initial value problem:

$$x y' + 2y = x^2 - x + 1 \ , \quad y(1) = \frac{1}{2}$$ (14 marks)

[TOTAL 25 MARKS]

Continued ...

QUESTION 3

a. Expand $\sqrt{4+3x}$ in ascending powers of x up to the term in x^3. Hence, by substituting an appropriate value of x, find the value of $\sqrt{4.3}$ correct to five decimal places. (11 marks)

b. Find the vector equation of the line that passes through points $A(1,2,7)$ and $B(2,-3,4)$. Hence, express the equation of the line in parametric and Cartesian form. (7 marks)

c. Given that the vector $a=\begin{pmatrix}3\\2\\1\end{pmatrix}$, $b=\begin{pmatrix}m\\-1\\n\end{pmatrix}$ and $c=\begin{pmatrix}5\\m-1\\3\end{pmatrix}$, where m and n are constants. If $a+b=c$, find the numerical values of m and n. Hence, find a unit vector in the direction of vector b and a vector of magnitude 6 units in the direction of vector b. (7 marks)

[TOTAL 25 MARKS]

QUESTION 4

a. Given that $z=\dfrac{x+y}{\sqrt{x^2+y^2}}$, find z_x and z_y. Hence, show that $xz_x+yz_y=0$.

(15 marks)

b. Evaluate the integral $\int_0^1 \int_0^{z^2} \int_0^3 y\cos(z^5)\,dxdydz$. (10 marks)

[TOTAL 25 MARKS]

End of Page.

APPENDICES: Formulae

Integration of common functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C \qquad \int \sec^2 x\, dx = \tan x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$\int \csc^2 x dx = -\cot x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left(\left[f(x)\right]^2 - \left[g(x)\right]^2\right) dx$$

$$V = \int_c^d \pi\left(\left[w(y)\right]^2 - \left[v(y)\right]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$

$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Linear Differential Equations:

$$\frac{dy}{dx} + p(x)y = q(x);\ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \qquad \text{where } \mu = e^{\int p(x) dx}$$

Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
p-series	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ If $0 < c < \infty$, then both series converge or both diverge.
Alternating Series Test	If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad b_n > 0$ Satisfies : i. $b_{n+1} \leq b_n$ for all n ii. $\lim_{n\to\infty} b_n = 0$ then the series is convergent
Ratio Test	Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$ a. Series converges absolutely if $L < 1$ b. Series diverges if $L > 1$ or $L = \infty$. c. No conclusion if $L = 1$

Binomial expansion

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Vector

The length of the vector $\mathbf{a} =< a_1, a_2, a_3 >$ is $|a| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$.

If θ is the angle between the vector **a** and **b**, then $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Cross Product $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$

Equation of Line

Vector equation: $\boldsymbol{r = r_0 + tv}$

Parametric equation: $\boldsymbol{x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct}$

Equation of Plane $< a, b, c >< x - x_0, y - y_0, z - z_0 >= 0$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a, b) is a critical point of f]. Let

$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.
b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum
c. If $D < 0$, then $f(a, b)$ is a saddle point.

Moments and Centers of Mass

The **moment** about the ***x*-axis:**

$$M_x = \iint_D y\rho(x,y)dA$$

The **moment** about the **y-axis:**

$$M_y = \iint_D x\rho(x,y)dA$$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\rho(x,y)dA \qquad \bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\rho(x,y)dA$$

Where the mass:

$$m = \iint_D \rho(x,y)dA$$

Triple Integrals: $\iiint_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)dxdydz$